3. Probabilistic voting

Voting and Electoral Competition
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Probabilistic voting
(only the main idea required in the exam)

- The use of median voter models is problematic when the policy space is multidimensional. If there is no policy location that dominates all others, there is always a possibility of cycling.

- One solution is provided by probabilistic voting models. In such models, the probability that a voter votes for a given candidate is a continuous function of policy positions.
Another solution is structure-induced equilibrium; see subsection 2.9

With probabilistic voting, candidates differ in their platform and in another dimension that could be ideology, perceived competence, likeability, looks etc. What is crucial is that the other dimension cannot be chosen by the candidate (party)

Furthermore, it is crucial that voters differ in their evaluation of this characteristics

In general, candidates target voter groups who are easy to buy. That is, voters who do not differ much in their evaluations of the intrinsic qualities of the two candidates
Additionally, there is a popularity shock parameter that varies between the competing parties.

Interestingly, probabilistic voting results in office-motivated candidates to converge fully in their platforms.

Example (Persson – Tabellini, p. 53-58)

- Three groups: rich (R), the middle-class (M) and poor (P)
- Everyone within one group has the same income, denoted by 
  \[ y^J, y^R > y^M > y^P \]
- Population shares sum to one:
  \[ \alpha^R + \alpha^M + \alpha^P = 1 \]
Voters take into account both economic policy and candidate ideology (this is best interpreted as other than economic ideology).

Voter $i$ in group $j$ prefers candidate A if

$$W^j(g_A) > W^j(g_B) + \sigma^{ij} + \delta$$

where $\sigma^{ij}$ is an individual-specific parameter that measures to what extent individual $i$ prefers candidate B. It can have also negative values which would indicate bias towards candidate A.

Parameter $\delta$ measures the average relative popularity of candidate B. It is assumed to follow a uniform distribution on

$$\left[ -\frac{1}{2\Psi}, \frac{1}{2\Psi} \right]$$

with density $\Psi$. 
As average popularity of candidate B is measured by a separate parameter, we can assume, without loss of generality, that the average value of individual-specific parameters is zero.

Furthermore, it is assumed that they follow group-specific uniform distributions on \[-\frac{1}{2\Phi J}, \frac{1}{2\Phi J}\]

with density \(\Phi\)

The timing of events:

1. candidates announce their electoral platforms, \(g_A, g_B\).

   ➢ They know the distributions for preference parameters, but not realizations

2. the realization of \(\delta\) is revealed

3. elections take place

4. the elected candidate implements his or her platform
To solve for the optimal behavior of office-motivated candidates, we first solve in each group the swing voter who is indifferent between the two candidates:

$$\sigma^I = W^I(g_A) - W^I(g_B) - \delta$$

All voters in group $J$ with $\sigma^I \leq \sigma^J$ prefer party $A$.

The fraction of votes that candidate $A$ receives in group $J$ is

$$F\left(\sigma^I + \frac{1}{2\phi^I}\right)$$

Therefore, the total vote share that candidate $A$ receives is

$$\pi_A = \sum_{j} \alpha^j \Phi^j \left(\sigma^I + \frac{1}{2\phi^I}\right)$$

Candidate $A$ wins if this is more than 0.5. Whether this is the case depends on the random realizations and on the chosen platforms.
This probability can be derived as follows.

- Probability that A wins is $p_A$; it satisfies:
  
  $p_A = \text{Prob}(\pi_A > 0.5)$
  
  $p_A = \text{Prob}(\sum_j \alpha^j \Phi^j (\sigma^j + \frac{1}{2\Phi^j}) > 0.5)$
  
  $p_A = \text{Prob}\left(\sum_j \alpha^j \Phi^j \left(W^j(g_A) - W^j(g_B) - \delta + \frac{1}{2\Phi^j}\right) > 0.5\right)$

The second line uses the definition of $\pi_A$ and the third line the definition of $\sigma^j$

Rearranging gives

$p_A = \text{Prob}\left(\sum_j \alpha^j \Phi^j \left(W^j(g_A) - W^j(g_B) + \frac{1}{2\Phi^j} - \frac{1}{2}\right) > \delta \sum_j \alpha^j \Phi^j \right)$

Next insert notation

$\Phi = \sum_j \alpha^j \Phi^j$

With this

$p_A = \text{Prob}\left(\sum_j \alpha^j \Phi^j \left(W^j(g_A) - W^j(g_B) + \frac{1}{2\Phi^j} - \frac{1}{2}\right) > \delta \Phi\right)$
It turns out that in equilibrium, both candidates choose the same policies.

- This result depends crucially on the assumption that candidates are office-motivated.
- With ideological motivations that could favour one of the groups, platforms would not converge.

In equilibrium, both candidates maximize a weighted sum of the welfare of the different voter groups.

- Weights of the groups depend on the density of their ideological distribution.

The more narrowly a given group is distributed ideologically, the more a politician can gain votes in that group by promising more expenditures benefiting it.

In equilibrium, politicians spend money so that the expected number of votes that an additional euro brings is the same in each group.

- This tends to benefit groups for which government spending is very important, like farmers and residents in poor regions that receive a lot of subsidies.
- Groups for which ideology is not important are easier to persuade by spending, and thus both politicians target such groups.