# Efficiency and Equity Aspects of Alternative Social Security Rules

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This paper studies human-capital formation, labor-supply, and retirement decisions associated with four alternative regimes of social security. We implement a theoretical model with overlapping generations of households and two different ability types within each generation. We find that with a given social security contribution rate, it is better to transfer income to the elderly as old-age benefits, paid independently of labor-market status. This holds with both Bismarckian and Beveridgean benefits. With sufficiently small ability differences, a Bismarckian system of old-age benefits is likely to offer the highest level of utility to all citizens.

*Keywords:* social security, education, retirement, labor supply, general-equilibrium models

*JEL classification:* C 68, H 55, I 21, J 22, J 26

## 1. Introduction

Broadly speaking, the European retirement schemes can be classified into *Bismarckian* systems, where earnings-related pensions are mainly financed by earnings-related contributions, and *Beveridgean* systems, characterized by tax-financed, flat-rate benefits based on the individual need to avoid poverty in old age. In the former system a constant replacement ratio is maintained across income levels; in the latter, replacement ratios are decreasing. France, Germany, and Italy follow the Bismarckian tradition, while the U.K., the Netherlands, and Denmark are examples of Beveridgean systems.[1]

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1 For a detailed description of the differences between pension systems in Europe, see Schmähl (1993).

We would expect these systems to differ in their economic effects, with respect to both efficiency and equity. For example, a Beveridgean system redistributes income from people with high earnings before retirement to people with low earnings, thereby creating a more equal income distribution. Though a Bismarckian system fails to satisfy the demand for reduced income inequality, originating from egalitarian values and/or political pressures, it may reduce existing distortions with respect to labor supply and human-capital investment. Therefore, a Bismarckian system may lead to a higher level of production at the expense of a less equal income redistribution. Thus, a society valuing both equity and efficiency is faced with a dilemma: According to which principle should social security systems be organized? What is the optimal balance between equity and efficiency? This paper provides analytical insights into both the qualitative and the quantitative aspects of this trade-off.

Another crucial feature of a social security system is whether there is an additional implicit tax on continued work after entitlement age. Recent evidence suggests that such implicit taxes are quite common in OECD countries and that they play a big role in retirement decisions (Gruber and Wise, 1999).[2] We call systems in which continued work after entitlement age does not reduce benefits *old-age benefits*. Such benefits are paid to those above entitlement age independently of their labor-market status.[3] We refer to the other type of benefits, namely those paid only on condition that the recipient has retired from the labor market, as *retirement subsidies*.[4] The institutional interpretation of retirement subsidies is as follows. Full retirement benefits are paid only to those individuals who do not work at all. The benefit is then scaled down for every additional hour worked. In other words, part-time retirement is allowed. Such part-time retirement programs, active in countries like Germany and the United States, aim at a smooth transition from labor-market participation to retirement. In summary, we analyze and compare four alternative systems of social security: (i) Beveridgean old-age benefits; (ii) Bismarckian old-age benefits; (iii) Beveridgean retirement subsidies; and (iv) Bismarckian retirement subsidies.

2 These taxes on continued work appear to be particularly large in Germany and France, and relatively small in the United States. For example, Börsch-Supan (2000) finds that retirement before age 60 could be reduced by more than a third in Germany if the social security system were made actuarially fair. With actuarial adjustment, the net present value of lifetime social security benefits does not depend on retirement behavior. Thus, we define actuarial fairness to be fairness at the margin. Gruber and Wise (1999) provide a useful survey of the link between social security and retirement in several countries.

3 The same effect can be achieved if benefit levels are increased by postponing retirement.

4 While “Bismarckian” and “Beveridgean” are widely used terms, there is no widely agreed terminology to refer to the two rules according to which benefits are paid after the entitlement age. What we call old-age benefits are actuarial at the margin, while what we call retirement subsidies are an example of a system that is not actuarial at the margin.

As our analytical framework, we introduce a general-equilibrium model of a closed economy with overlapping generations. The analysis is based on a steady-state equilibrium under standard assumptions, including intertemporal utility maximization, perfect foresight, and perfect competition in each market. Households make decisions over time about labor supply, human-capital investment, and retirement. We mean by retirement a permanent reduction in the share of time devoted to working. We view retirement as a continuous choice, taking into account that the elderly may continue part-time work even after they become entitled to social security benefits. Our model also allows individuals to start reducing their labor supply before they are entitled to social security benefits.

Inside each cohort there are two different ability classes, defined by differences in the productivity of human capital.[5] With any given stock of human capital and time devoted to work, individuals with higher ability are able to supply more efficiency units of labor. We focus on the effect that alternative social security systems have on retirement decisions and human-capital investment.[6] Such analysis provides insights into long-term effects in case a country were to reform its social security system. This is especially relevant in the European context, both due to population aging and because European integration may lead to convergence in the systems of social protection. Indeed, it would be useful to know if we are "picking the winner" in case just one scheme survives a competition. In order to answer that question we need to identify relevant alternatives among current systems, an appropriate model, and some assessment criteria. We provide analytical tools to evaluate the distributional and efficiency issues involved.

Previous literature has devoted considerable attention to the effects of changing the entitlement age. For example, Cremer and Pestieau (2003) argue that increasing the age of entitlement to retirement benefits could generate a double dividend. First, it would free resources needed to meet the challenge of population aging. Second, it could improve the lifetime welfare of those with low wages and poor health in countries with redistributive social security schemes. Cremer and Pestieau (2003) identify as necessary conditions for a double dividend that the benefit rule must both redistribute

5 We extend the model in Lau and Poutvaara (2001) by incorporating different ability types instead of just one. We are not aware of any other papers that analyze the effects of alternative social security rules with endogenous human capital in a computational general-equilibrium model, thus aiming at evaluating also the magnitude of incentive effects.

6 The role of endogenous human-capital accumulation has mainly been studied in the context of tax reforms, where computational general-equilibrium models have been used to evaluate the effects of replacing a comprehensive flat income-tax system with progressive wage-income taxation and proportional capital-income taxation (Heckman et al., 1998) or taxation based on consumption (Perroni, 1995).

within generations and induce early retirement. Our paper complements their analysis by studying the effects of changes in benefit rules with a given entitlement age. We find that a system that does not induce early retirement is more efficient and results in a higher welfare for both high- and low-ability types. If the social security scheme is planned efficiently with any given entitlement age, there is no longer scope for a double dividend. Of course, this result in itself does not suggest that the entitlement age should not be increased, but only that a Pareto improvement may not be possible.

The paper proceeds as follows. Section 2 outlines an economy with Beveridgean old-age benefits, chosen as benchmark, and it includes a characterization of the steady state and the calibration method. Section 3 then provides a formal presentation of each alternative social security system. Section 4 demonstrates our results, and concluding remarks are found in section 5.

## 2. The Benchmark Model

Our starting point is a Beveridgean social security system with old-age benefits. Labor income taxes are collected to satisfy a given revenue requirement, while social security benefits are financed by a separate payroll tax. In the following we set out the details of this framework.

### 2.1. Intertemporal Optimization

In each period of the life cycle, individuals divide time between work ($q$), learning ($s$), and leisure ($v$). The economic life span of each individual consists of 60 periods, each period representing one year, and the periods are indexed from 0 to 59. The size of each cohort is normalized at 2, implying that the size of the population is equal to 120. The total use of time in each period cannot exceed the time endowment, and the following constraint on time applies:

$$v_{i,t} + q_{i,t} + s_{i,t} \leq e, \tag{1}$$

where $e$ is the constant endowment of time in each period. The time endowment comprises hours available for work, learning, and leisure, and it is therefore interpreted as the normal length of a work week, say 40 hours. Subscript $t$ refers to the citizen's age, and subscript $i$ to the ability type.

Gross investment in human capital is determined by learning, and the stock of human capital evolves according to the following law of motion:

$$h_{i,t+1} = (1 - \delta^H)h_{i,t} + s_{i,t}^{\eta}, \tag{2}$$

where $h_{i,t}$ is the capital stock of an individual of ability type $i$ in period $t$, $\delta^H$ is the rate of depreciation of human capital, and $\eta$ measures the elasticity of new human capital with respect to time devoted to learning, with $0 < \eta \leq 1$. The initial stock of human capital, $h_0$, is positive, and the economic agents thus possess some productive skills without learning effort. Each individual has the same initial stock of human capital, but subsequent levels of human capital diverge due to differences in ability and time devoted to learning.

The effective supply of labor services depends on individual ability, time devoted to work, and the stock of human capital:

$$l_{i,t} = a_i q_{i,t} h_{i,t}^{\beta}, \tag{3}$$

where $a_i$ is an individual-specific ability parameter, $l_{i,t}$ is the individual supply of labor services, and $\beta$ is the elasticity of productivity with respect to the stock of human capital. There are two ability types: high and low. In each cohort, the mass of individuals of both ability types is 1. The low-ability type $a_l$ and high-ability type $a_h$ are chosen so that $(a_l + a_h)/2 = 1$. We measure relative ability differences between high and low ability types by

$$d = \frac{a_h - a_l}{a_l}.$$

We assume that $0 < \beta < 1$, implying diminishing marginal productivity of human capital in the production of effective labor services.

The individual maximization problem for an individual of ability type $i$ is based on an explicit representation of the utility function:

$$U_i = \sum_{t=0}^{59} \frac{1}{(1+\rho)^t} \left( \frac{c_{i,t}^{1-\gamma}}{1-\gamma} + \alpha\left(v_{i,t} - \mu v_{i,t}^2\right) \right), \tag{4}$$

where $c_{i,t}$ is the consumption of goods in period $t$, $v_{i,t}$ is the demand for leisure in period $t$, $\rho$ is the rate of time preference, $\alpha$ is a weight parameter for leisure, $\gamma$ is the reciprocal of the intertemporal elasticity of substitution $\sigma$, and $\mu$ reflects the rate at which the marginal utility of leisure decreases as the amount of leisure is increased. The specification of the instantaneous utility function implies that individuals smooth consumption across the life cycle, whereas leisure is not necessarily consumed in every period. The marginal utility from leisure is $\alpha(1 - 2\mu v_{i,t})$, implying a constant marginal utility if $\mu$ equals zero. A natural upper bound for $\mu$ is $\frac{1}{2e}$, i.e., the marginal utility of leisure goes to zero if all the time endowment is spent on leisure.

Contrary to popular constant-elasticity-of-substitution formulations of the utility function, the additively separable utility function allows the point in time at which the individual begins to retire to be endogenous. Retirement begins when the individual starts to demand a positive amount of leisure. Demanding a positive amount of leisure means that less time than the normal work week is devoted to work and human-capital formation.

The quadratic utility function for leisure allows us to capture two realistic features of actual labor markets. First, active participation in the labor market is phased out at old age. People typically study or work full time when they are young[7] because the period during which the returns on human-capital investment can be collected is long. After investment in human capital is phased out, productivity starts to decline and the opportunity cost of leisure is reduced. Second, people typically value a marginal increase in leisure more when leisure time is scarce.

Individuals are born without financial wealth, and they can save and borrow without liquidity constraints at the market interest rate, $r$. The price of the consumption good is chosen as numeraire and normalized at unity. The lifetime budget constraint states that the present value of lifetime expenditures on consumption cannot exceed the present value of lifetime wage income and retirement benefits:

$$\sum_{t=0}^{59} \frac{W}{(1+r)^t} \cdot l_{i,t} + \sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \kappa \geq \sum_{t=0}^{59} \frac{1}{(1+r)^t} \cdot c_{i,t}, \tag{5}$$

where $W$ is the net return to labor services, and $\kappa$ is the constant social security payment per period to individuals above the entitlement age. Each individual maximizes the present value of lifetime utility, $U$, subject to the time endowment constraint, the law of motion with respect to human capital, and the intertemporal budget constraint.

## 2.2. General Equilibrium

Given the assumption that all individuals are identical inside the two ability classes, it is a simple task to derive the aggregate supply of labor services to the labor market and consumption in the steady state. Since the size of each cohort is constant and equal to 2, the aggregate supply of labor services is equal to the sum of the supply of labor services over the life cycle for the representatives of the two ability types:

$$L = \sum_{i=1}^{2} \sum_{t=0}^{59} l_{i,t}, \tag{6}$$

where $L$ is the aggregate supply of labor services in steady state and $l_{i,t}$ is the effective supply of labor services by the citizens of age $t$ and ability type $i$.

The production of final goods combines labor services and physical capital, and the technology is represented by a Cobb–Douglas specification:

$$Y = K^{\phi} L^{1-\phi}, \tag{7}$$

7 This is a reasonable assumption. Indeed, Friedberg (2003) finds strong evidence of forward-looking factors in the decision to acquire computer skills: the number of years to retirement is shown to be an important determinant, which indicates that the length of the remaining working life plays a role in skill acquisition.

where $Y$ is the aggregate output, $K$ is the aggregate stock of physical capital, and $\phi$ is the value share of physical capital. Each producer of goods maximizes profits subject to the production technology, and the first-order conditions imply that the marginal product of the particular factor input is equal to the producer price of that factor input.

The capital stock in period $t$ is equal to the capital stock at the beginning of the previous period less depreciation plus investment in the previous period. The capital stock is constant in the steady state, and the gross investment in physical capital is thus given by

$$I = \delta^K K, \tag{8}$$

where $I$ is gross investment in physical capital, and $\delta^K$ is the rate of depreciation of physical capital.

Aggregate output is either invested or consumed by individuals and the public sector, and the market-clearing condition for goods is

$$Y = C + I + G, \tag{9}$$

where $C$ is aggregate private demand for consumption goods, and $G$ is public demand for goods.

The government operates with two dynamic budget constraints: one constraint on social security expenditures and another constraint on other public expenditures. Both budgets are balanced in each period. The constraint with respect to social security is

$$\tau^s WL = \sum_{i=1}^{2} \sum_{t=45}^{59} \kappa_{i,t}, \tag{10}$$

where $\tau^s$ is the social security tax rate. The social security tax payment is defined in relation to after-tax wage income. The left-hand side is equal to the social security tax revenue, and the right-hand side is equal to social security expenditures to entitled generations. On the right-hand side, the summation is carried out over both ability types for each entitled age group. In this example, we assume that individuals are eligible for old-age benefits at age 63.[8] The budget constraint with respect to the supply of other public goods and services is

$$\tau^l WL = G, \tag{11}$$

where $\tau^l$ is the tax rate on labor income. The tax rate on labor income is also defined in relation to the after-tax wage income.[9]

8 In this paper, we do not analyze reforms that would entail increasing the entitlement age. If the level of benefits per year of retirement were kept constant, such reforms would effectively scale down the social security system. This would benefit all steady-state generations at the expense of the elderly losing benefits during the transition.

9 The corresponding net tax rates are determined by $\tau^s/(1+\tau^l+\tau^s)$ and $\tau^l/(1+\tau^l+\tau^s)$.

## 2.3. Welfare Effects

We use the equivalent variation measure to assess private welfare effects of the social security reform. This measure is derived as the percentage change in lifetime earnings (in base-year prices) necessary to yield the welfare level reached in the new steady state. More formally, the equivalent variation is determined by

$$U_i(E_{i,0}(1+\chi), r_0, w_0) = U(E_{i,1}, r_1, w_1), \tag{12}$$

where $E_i$ is the net present value of lifetime income for an individual of ability type $i$, and the subscripts 0 and 1 denote the initial and the new steady state, respectively. The parameter $\chi$ measures the change in private welfare between the two steady states. This welfare measure can be compared across different steady states and is applicable for changes of any size and not only differential approximations.[10] The equivalent variation measure is calculated separately for each ability class. Finally, we also report a utilitarian welfare measure based on the sum of utility for both ability types.

## 2.4. Calibration

The model is calibrated to the data set presented in table 1, and we apply the following standard parameter values in the baseline scenario. Capital and labor income account for, respectively, 30% and 70% of GDP, implying that the labor–capital income ratio is equal to 2.3. The level of investment is equal to 20.1% of GDP, given a net interest rate of 3.5% and a 10% depreciation rate of physical capital. To achieve a sufficiently high private saving rate, the rate of time preference is set equal to 1.5%.[11] We assume that the intertemporal elasticity of substitution is equal to 0.667, which is within the range from 0.5 to 1 that is applied in most numerical studies. The parameters in the utility function and in the production technology for human capital are chosen to generate lifetime labor-supply profiles that correspond qualitatively to those estimated by McGratten and Rogerson (1998).

Lau et al. (forthcoming) use the same framework as in this paper to evaluate a different policy question, namely the efficiency effects of a military draft in the presence of human-capital formation. Their sensitivity analysis suggests that the qualitative results are robust to even wide changes in the parameter values in the production function of human capital. In the sensitivity

10 The problem is formally solved by minimizing the change in lifetime income subject to $U_0 = U_1$ and the constraints in the individual maximization problem.

11 Although recent empirical evidence suggests that individual discount rates vary with household characteristics such as income (Harrison et al., 2002), we apply the standard assumption that the rate of time preference is constant and the same for all household types.

**Table 1**
*Parameter Values in the Initial Steady State*

| | | |
|---|---|---|
| *Parameter values:* | | |
| $\rho$ | Time preference rate | 0.015 |
| $\sigma$ | Intertemporal elasticity of substitution | 0.667 |
| $\mu$ | Coefficient in quadratic utility function | 0.300 |
| $\alpha$ | Weight parameter in quadratic utility function wrt. leisure | 0.960 |
| $\eta$ | Elasticity of human capital wrt. time devoted to learning | 0.750 |
| $\delta^H$ | Depreciation rate for human capital | 0.100 |
| $\beta$ | Elasticity of labor services wrt. human capital | 0.350 |
| $\phi$ | Value share of physical capital in production of goods | 0.305 |
| $\delta^K$ | Depreciation rate for physical capital | 0.100 |
| $h_0$ | Initial human capital stock | 1.000 |
| $e$ | Endowment of time in each period | 1.000 |
| $d$ | Relative ability difference | 0.500 |
| *Factor prices:* | | |
| $r$ | Annual interest rate | 0.035 |
| $w$ | Wage rate after tax | 1.000 |
| *Tax rates:* | | |
| $\tau^l$ | Tax rate on labor income | 0.430 |
| $\tau^s$ | Social security tax rate | 0.222 |
| *Public expenditures:* | | |
| Social security expenditures / *GDP* | | 0.093 |
| Public spending on goods and services / *GDP* | | 0.181 |

analysis in this paper we focus on the parameters in the utility function, and report results with a wide distribution of relative ability difference between high-ability and low-ability types.

The gross tax rate on wage income in the initial steady state is 43% and covers public expenditures on goods and services, which are equal to 18.1% of GDP. The social security tax rate is 22.2%, and public expenditure on social security is 9.3% of GDP. These public expenditure levels are not out of the ordinary in industrialized countries.

As to the allocation of time, both ability types follow the same pattern. Time is divided between learning and work effort in the first part of the life cycle and between work effort and leisure in the second part. The individual labor-supply curves resemble the estimated labor-supply profile for recent generations in McGratten and Rogerson (1998), viz., the time spent on work increases sharply during the first five years of the life cycle, is constant during middle age, and begins to fall when individuals begin to retire from the labor market.[12]

## 3. Alternative Social Security Systems

This section characterizes three alternative social security systems: Bismarckian old-age benefits, Beveridgean retirement subsidies, and Bismarckian retirement subsidies.[13] Beveridgean and Bismarckian systems differ crucially, in efficiency as well as equity effects. Beveridgean systems implement direct income redistribution from high-ability to low-ability citizens. In principle, a Bismarckian old-age benefit system does not allow for income redistribution, because benefits are defined as a constant fraction of past contributions.[14] In practice, however, even a Bismarckian old-age benefit system may have small redistributive effects, as social security contributions collected after the entitlement age do not influence the benefit level. A Bismarckian retirement subsidy system allows for income redistribution between different ability types if time allocation patterns differ. The direction of income redistribution is *ex ante* unclear. If the low-ability individuals spend more time in retirement, then they benefit from income redistribution. However, if they spend less time in retirement, then they are actually cross-subsidizing higher benefits to the high-ability types.

Old-age benefit systems can be expected to yield more efficient labor-supply decisions over the life cycle, because retirement subsidy systems subsidize retirement and reduce the supply of labor after the entitlement age. Bismarckian systems introduce an intertemporal distortion in the supply of labor, since social security benefits depend on wage income before and not after the entitlement age. On the other hand, the private return to labor supply before the entitlement age is closer to the market return, and the Bismarckian system may therefore reduce the distortion due to the labor income tax. To evaluate the combined effect of these opposite forces and compare alternative social security rules, the calibrated parameter values are kept unchanged across the three scenarios. Public provisions of goods and social security are kept unchanged, and the social security tax rate and

12 All relevant first-order conditions for this and other systems are reported in the appendix.

13 Our representation of retirement subsidy systems implies that pensions are cut to the extent that people continue with part-time work.

14 Old-age benefit systems implement actuarial adjustment at the margin. From an individual's perspective, the timing of retirement after the entitlement age does not change the value of his/her social security wealth.

the tax rate on wage income are determined endogenously to balance the public budget.[15]

### 3.1. Bismarckian Old-age Benefits

Under the Bismarckian old-age benefit system, the individual intertemporal budget constraint changes to

$$\sum_{t=0}^{59} \frac{W}{(1+r)^t} \cdot l_{i,t} + \sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \pi_i \geq \sum_{t=0}^{59} \frac{1}{(1+r)^t} \cdot c_{i,t}, \tag{13}$$

where $\pi$ is the earnings-related social security payment per period to individuals after the entitlement age. Old-age benefits depend on the wage income history during the period before the entitlement age and are determined by

$$\sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \pi_i = x \cdot \sum_{t=0}^{44} \frac{W}{(1+r)^t} \cdot l_{i,t}, \tag{14}$$

where $x$ is a positive number, which is determined by the government in accordance with the public budget constraint. This specification implies that the net present value of old-age benefits during the official retirement period is a constant fraction of the net present value of after-tax wage income before the entitlement age.

The government budget constraint with respect to the social security system changes to

$$\tau^s WL = \sum_{i=1}^{2} \sum_{t=45}^{59} \pi_i, \tag{15}$$

where the left-hand side is the social security tax revenue, and the right-hand side is the aggregate social security payment to current old generations. The responsiveness parameter is determined by this constraint and the above link (14) between contributions and benefits.

### 3.2. Beveridgean Retirement Subsidies

Under the Beveridgean retirement subsidy system, the individual intertemporal budget constraint changes to

$$\sum_{t=0}^{59} \frac{W}{(1+r)^t} \cdot l_{i,t} + \sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \omega v_{i,t} \geq \sum_{t=0}^{59} \frac{1}{(1+r)^t} \cdot c_{i,t}, \tag{16}$$

where $\omega$ is the retirement subsidy. We assume that hours worked are observable, corresponding to the idea of allowing for part-time retirement. The

15 We use GAMS/MPSGE to solve the model numerically. Rutherford (1999) documents this modeling system and software.

social security budget constraint now reads

$$\tau^s WL = \sum_{i=1}^{2} \sum_{t=45}^{59} \omega v_{i,t}, \tag{17}$$

where the left-hand side is the social security tax revenue, and the right-hand side is the aggregate social security payment to current old generations.

### 3.3. Bismarckian Retirement Subsidies

The Bismarckian retirement subsidy system includes a link between social security contributions and benefits, where benefits after the entitlement age depend linearly on the wage income before that age. The individual budget constraint is determined by

$$\sum_{t=0}^{59} \frac{W}{(1+r)^t} \cdot l_{i,t} + \sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \psi_i v_{i,t} \geq \sum_{t=0}^{59} \frac{1}{(1+r)^t} \cdot c_{i,t}, \tag{18}$$

where $\psi_i$ is the income-dependent subsidy to retirement after the entitlement age for ability type $i$. The social security benefit stream is equal to a fraction of the net present value of net wage income before the entitlement age:

$$\psi_i = z \cdot \sum_{t=0}^{44} \frac{W}{(1+r)^t} \cdot l_{i,t}, \tag{19}$$

where the fraction $z > 0$ is given to all individuals and determined by the government in accordance with the public budget constraint. $z$ measures the responsiveness of retirement subsidy to past earnings.

The government budget constraint with respect to the social security system changes to

$$\tau^s WL = \sum_{i=1}^{2} \sum_{t=45}^{59} \psi_i v_{i,t}, \tag{20}$$

where the left-hand side is the social security tax revenue, and the right-hand side is the aggregate social security payment to current old generations. The responsiveness parameter $z$ is determined by the government budget constraint and (19) taken together.

## 4. Results

This section presents the results of comparing the four alternative social security rules. We first demonstrate how the rules differ in their macroeconomic effects, and then provide a welfare comparison.

## 4.1. Macroeconomic Effects

### 4.1.1. Time Allocation

Economic intuition suggests that human-capital investment is concentrated in the early years of life, and retirement takes place at a later stage. Indeed, the expected period to reap benefits from learning is reduced towards the end of life, thereby reducing the incentives to maintain the human-capital stock. The depreciation of human capital reduces the wage rate and leads to a fall in the opportunity cost of leisure. However, it is not *a priori* clear how the retirement pattern is related to ability. The income effect tends to induce high-ability citizens to retire earlier in order to consume more leisure, whereas the substitution effect would encourage them to work more and therefore to retire later.

Our numerical results suggest that the income effect dominates with the functional forms analyzed, i.e., high-ability people retire earlier than low-ability people.[16] This result turns out to be robust to variations in the weight on leisure in the utility function, the rate at which the marginal utility from leisure is diminishing, and the size of the ability cap. To reconcile our results with empirical evidence suggesting that low-income workers often retire earlier, it is appropriate to highlight two features omitted from our model. First of all, low-income workers often work in physically more strenuous occupations. Therefore, they may need to retire earlier due to health concerns. Secondly, high-income jobs are often more remunerative also in nonmonetary aspects. While we acknowledge the importance of these concerns, including such considerations would blur the effects of social security rules on labor supply, human-capital investment, and retirement, which are our focus.

Our results also show that social security systems have a substantial effect on the allocation of time over the life cycle. Retirement subsidy systems tend to encourage *full* retirement when the entitlement age is reached, because social security benefits lower the private opportunity cost of leisure by the amount of the subsidy. Old-age benefit systems, on the other hand, encourage a more gradual retirement profile. Since the reduction in labor supply occurs earlier, old-age benefit systems also induce a more gradual reduction in human-capital investment.

We illustrate the effects of social security systems when the productivity parameter of high-ability types is 50% higher than that of low-ability types. Figures 1 and 2 depict the time devoted to learning by low-ability and high-ability types, respectively. There is no discernible difference in the time

16 Cremer and Pestieau (2003) analyze a two-period OLG model in which time used to work in the second period causes a quadratic loss, which is deducted from resources available for consumption. Their quasilinear utility function yields a result opposite to ours, namely, that the retirement age increases with the wage rate.

**Figure 1**
*Learning for Low-Ability Types ($d = 0.5$; $\mu = 0.3$; $\alpha = 0.96$)*

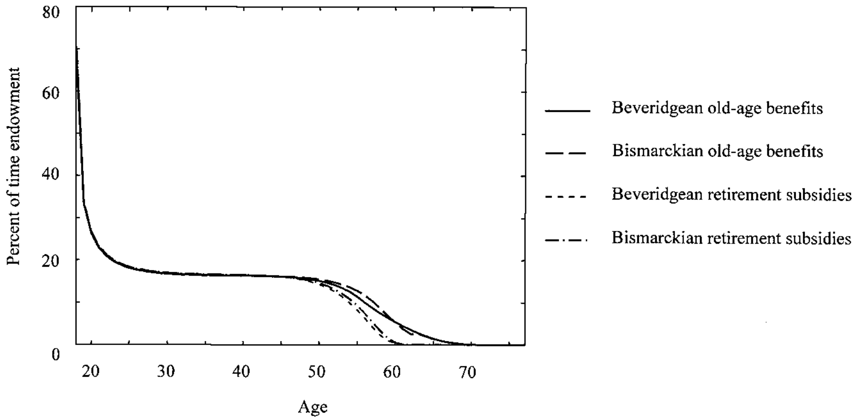
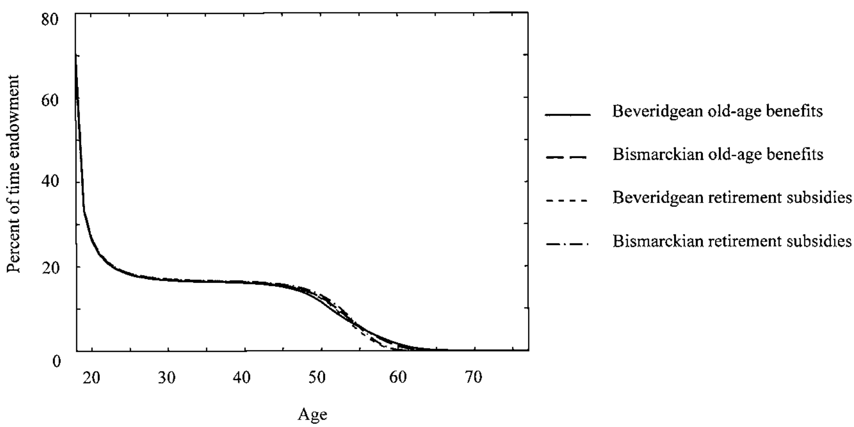



**Figure 2**
*Learning for High-Ability Types ($d = 0.5$; $\mu = 0.3$; $\alpha = 0.96$)*




devoted to learning by the two ability types before age 40, independently of the social security system in place. Low-ability types start reducing the time devoted to learning around the age of 50, while high-ability types start phasing out learning in their mid-forties.[17]

17 We assume that there is no difference in the learning technology used by the different ability types.

**Figure 3**
*Retirement for Low-Ability Types ($d = 0.5$; $\mu = 0.3$; $\alpha = 0.96$)*

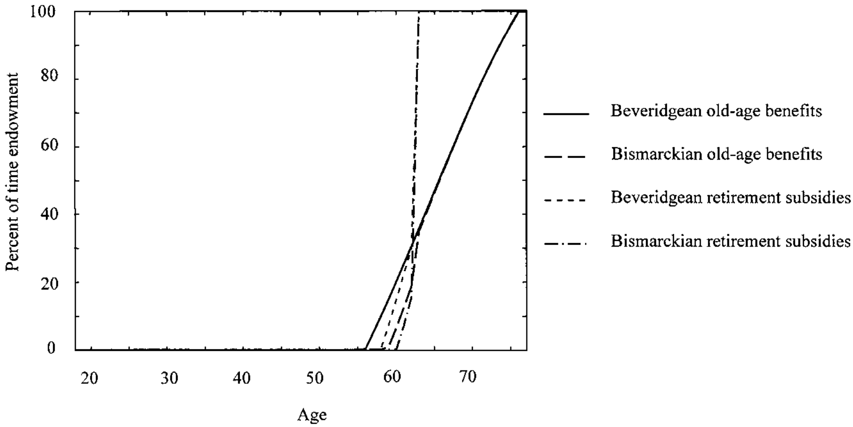
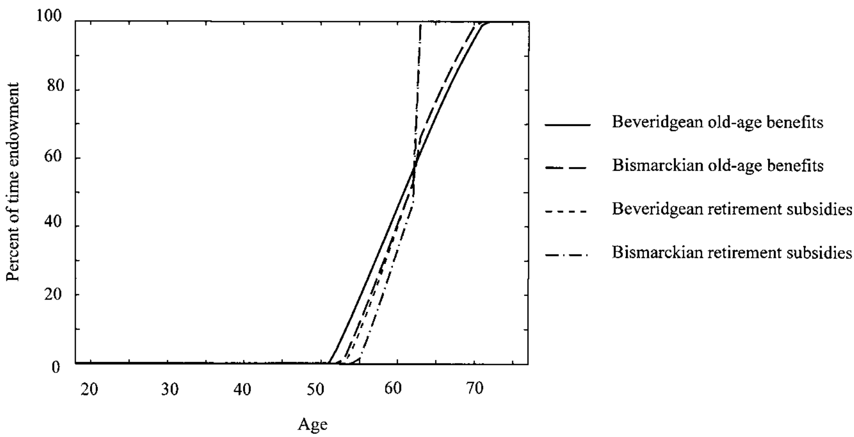



**Figure 4**
*Retirement for High-Ability Types ($d = 0.5$; $\mu = 0.3$; $\alpha = 0.96$)*




The effects of the social security systems differ between the two ability types. For low-ability types, old-age benefit systems encourage learning to continue until the age of 70, while with retirement subsidy systems learning ends by the age of 60. For high-ability types, learning is more gradually phased out. There is no significant difference in overall time devoted to learning across the social security systems, so a more gradual reduction in

**Figure 5**
*Labor Productivity for Low-Ability Types ($d = 0.5$; $\mu = 0.3$; $\alpha = 0.96$)*

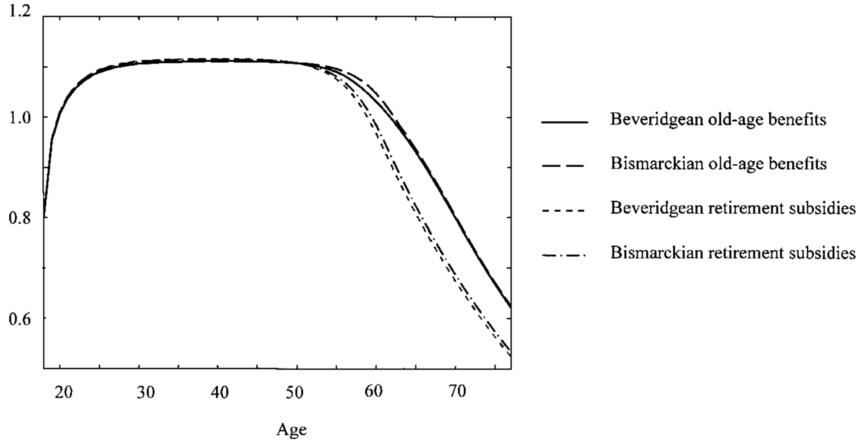
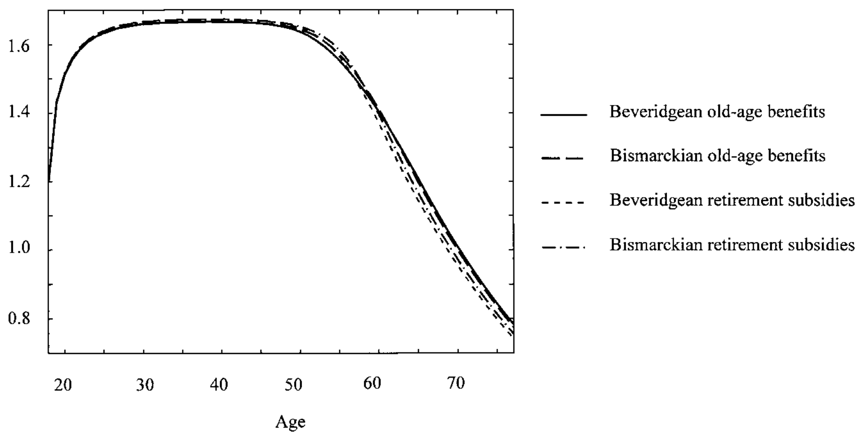



**Figure 6**
*Labor Productivity for High-Ability Types ($d = 0.5$; $\mu = 0.3$; $\alpha = 0.96$)*




learning under old-age benefit systems is associated with more learning later in the life cycle than in retirement subsidy systems.

Despite lower human-capital investment by high-ability types, their productivity never falls below that of low-ability types.[18]

18 This was checked for all combinations reported in tables 4–6. These results, as well as others reported only verbally, are available upon request.

The effects of social security systems on retirement differ markedly across old-age benefit and retirement subsidy systems and ability types. Figures 3 and 4 show that retirement subsidies encourage full retirement at the entitlement age of 63 for both ability types. With old-age benefits, low-ability types continue to work until the end of life, while high-ability types fully retire around the age of 70. Old-age benefit systems, on the other hand, lead to earlier retirement, but the profile is more gradual. Low-ability types start retirement at the age of 56 with Beveridgean old-age benefits, and at the age of 59 with Bismarckian old-age benefits. High-ability types start retirement at the age of 52 with Beveridgean old-age benefits, and at the age of 53 with Bismarckian old-age benefits.

Social security rules have quite small effects on the wage profile of high-ability types, while they have significant effects on wage profiles of low-ability types. Retirement subsidies result in a faster and more pronounced decline in the productivity of low-ability types after the age of 55. Rather small differences in time allocation patterns in figures 1 and 2 accumulate into quite large differences in wage profiles. Note that the scale is different in figures 5 and 6: even though the two ability types have roughly similar shape in their wage profile, high-ability types have higher wages over their whole life.

It is instructive to compare our results against a benchmark without ability differences ($d = 0$). In that case (not shown) the social security system turns out to have only a minor effect on learning when the representative individual is between 45 and 55 years old. However, we show that with ability differences, the effects of the social security system on learning may be considerable for low-ability types.

#### 4.1.2. Production and Consumption

Tables 2–4 show that the degree of ability differences in the labor market may influence the impact of social security reforms on production. We find that the highest production level is obtained by the Bismarckian old-age benefit system, whereas the lowest production level is obtained by the Beveridgean retirement subsidy system. Depending on parameter values, the difference in steady-state production between the Bismarckian old-age benefit system and the Beveridgean retirement subsidy system is between 0.9% and 4.3%. The relative position of the Beveridgean old-age benefit system and the Bismarckian retirement subsidy system varies. Furthermore, the loss caused by benefits being conditional on retirement increases when ability differences are high. This suggests that the cost of maintaining a retirement subsidy system would increase if the wage distribution became wider due to, say, skill-biased technological change.

**Table 2**

*Income Effects of Move to Alternative Social Security System when $\mu = 0.3$ and $\alpha = 0.96$ (percentage change from initial steady-state equilibrium)*

| | *Social security system* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Beveridgean retirement subsidies | | | Bismarckian old-age benefits | | | Bismarckian retirement subsidies | | |
| | *Ability difference* | | | *Ability difference* | | | *Ability difference* | | |
| | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 |
| GDP | −3.1 | −3.1 | −3.0 | 1.2 | 1.2 | 1.1 | −1.8 | −1.9 | −1.9 |
| Consumption | −4.7 | −4.7 | −4.7 | 1.7 | 1.6 | 1.6 | −2.8 | −3.0 | −3.2 |
| Physical-capital stock | −1.2 | −1.2 | −1.0 | 0.9 | 0.9 | 0.8 | −0.4 | −0.4 | −0.3 |
| Labor services | −3.8 | −3.9 | −3.9 | 1.3 | 1.3 | 1.3 | −2.4 | −2.5 | −2.7 |
| Rental rate on physical capital | −1.8 | −1.9 | −2.0 | 0.3 | 0.3 | 0.4 | −1.4 | −1.5 | −1.6 |
| Wage rate | 0.8 | 0.8 | 0.9 | −0.1 | −0.1 | −0.2 | 0.6 | 0.7 | 0.7 |
| Tax rate on labor income[a] | 3.5 | 3.5 | 3.4 | −1.3 | −1.2 | −1.2 | 2.0 | 2.1 | 2.2 |
| Equivalent variation for low ability | −2.25 | −2.24 | −2.22 | 0.35 | −0.93 | −2.47 | −1.82 | −3.29 | −5.08 |
| Equivalent variation for high ability | −2.27 | −2.31 | −2.34 | 0.99 | 1.83 | 2.45 | −1.14 | −0.33 | 0.22 |

Notes: (a) Change in percentage points.

**Table 3**

*Income Effects of Move to Alternative Social Security System when $\mu = 0.1$ and $\alpha = 0.74$ (percentage change from initial steady-state equilibrium)*

| | *Social security system* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Beveridgean retirement subsidies | | | Bismarckian old-age benefits | | | Bismarckian retirement subsidies | | |
| | *Ability difference* | | | *Ability difference* | | | *Ability difference* | | |
| | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 |
| GDP | −1.0 | −1.3 | −1.7 | 0.6 | 1.1 | 1.3 | −0.3 | −0.4 | −0.6 |
| Consumption | −1.4 | −1.9 | −2.6 | 0.9 | 1.5 | 1.9 | −0.4 | −0.7 | −1.0 |
| Physical-capital stock | −0.5 | −0.6 | −0.6 | 0.4 | 0.6 | 0.8 | −0.1 | −0.1 | 0.0 |
| Labor services | −1.2 | −1.6 | −2.1 | 0.8 | 1.2 | 1.5 | −0.3 | −0.6 | −0.8 |
| Rental rate on physical capital | −0.4 | −0.7 | −1.0 | 0.3 | 0.4 | 0.5 | −0.2 | −0.3 | −0.6 |
| Wage rate | 0.2 | 0.3 | 0.5 | −0.1 | −0.2 | −0.2 | 0.1 | 0.2 | 0.3 |
| Tax rate on labor income[a] | 1.0 | 1.4 | 1.8 | −0.7 | −1.1 | −1.4 | 0.3 | 0.5 | 0.6 |
| Equivalent variation for low ability | −0.61 | −0.86 | −1.17 | 0.04 | −1.04 | −2.46 | −0.55 | −2.09 | −4.01 |
| Equivalent variation for high ability | −0.16 | −0.86 | −1.17 | 0.69 | 1.79 | 2.57 | 0.12 | 0.85 | 1.31 |

Notes: (a) Change in percentage points.

**Table 4**

*Income Effects of Move to Alternative Social Security System when $\mu = 0.05$ and $\alpha = 0.70$ (percentage change from initial steady-state equilibrium)*

| | *Social security system* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Beveridgean retirement subsidies | | | Bismarckian old-age benefits | | | Bismarckian retirement subsidies | | |
| | *Ability difference* | | | *Ability difference* | | | *Ability difference* | | |
| | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 |
| GDP | –0.5 | –1.3 | –1.7 | 0.4 | 1.2 | 1.3 | –0.2 | –0.2 | –0.5 |
| Consumption | –0.8 | –1.9 | –2.6 | 0.5 | 1.7 | 1.9 | –0.3 | –0.3 | –0.9 |
| Physical-capital stock | –0.3 | –0.6 | –0.6 | 0.2 | 0.7 | 0.8 | –0.1 | 0.0 | 0.0 |
| Labor services | –0.7 | –1.6 | –2.1 | 0.4 | 1.4 | 1.6 | –0.2 | –0.3 | –0.8 |
| Rental rate on physical capital | –0.2 | –0.7 | –1.0 | 0.2 | 0.5 | 0.5 | –0.1 | –0.2 | –0.6 |
| Wage rate | 0.1 | 0.3 | 0.5 | –0.1 | –0.2 | –0.2 | 0.0 | 0.1 | 0.3 |
| Tax rate on labor income[a] | 0.6 | 1.4 | 1.8 | –0.4 | –1.2 | –1.4 | 0.2 | 0.2 | 0.6 |
| Equivalent variation for low ability | –0.34 | –0.83 | –1.16 | –0.12 | –0.99 | –2.46 | –0.48 | –1.94 | –3.98 |
| Equivalent variation for high ability | –0.34 | –0.83 | –1.18 | 0.54 | 1.86 | 2.59 | 0.19 | 1.00 | 1.34 |

Notes: (a) Change in percentage points.

The effects on consumption are even more varied. The difference in steady-state consumption between the system of Bismarckian old-age benefits and that of Beveridgean retirement subsidies varies between 1.3% and 6.4%. We also find that the effects of the social security system on steady-state production, the total supply of labor services, and the physical capital stock are correlated. A system that results in lower steady-state production leads also to a lower level of consumption, physical capital stock, and overall supply of labor services.

## 4.2. Welfare Comparisons

### 4.2.1. Equivalent Variation

Our framework allows us to rank the social security systems using a simple welfare measure applied to both ability types. As the basis of comparison, we have used equivalent variation. We find that high-ability types always prefer Bismarckian old-age benefits, while Beveridgean retirement subsidies result in the lowest level of welfare for that group. We also find that both ability types always prefer Beveridgean old-age benefits to Beveridgean retirement subsidies, and Bismarckian old-age benefits to Bismarckian retirement subsidies. Therefore, a nonactuarial arrangement of retirement benefits at the margin is not desirable. Old-age benefit systems are more desirable when the rate at which the marginal utility from leisure decreases is high. In some cases, that the system is that of old-age benefits becomes more important than income dependence for high-ability types, inducing them to prefer a Beveridgean old-age benefit system to a Bismarckian retirement subsidy system.

Our conclusion of the importance of actuarial adjustment is supported by Cremer and Pestieau (2003). They show that delaying retirement by three years would alone suffice to restore financial balance of the Belgian social security system. If retirement age and contribution rates were left unchanged, then restoring actuarial balance would require a 21% cut in the benefits.

The most surprising results concern the welfare ordering of alternative social security systems from the perspective of low-ability types. If the ability differences are not too large, low-ability types may actually prefer Bismarckian old-age benefits to Beveridgean old-age benefits. In other words, efficiency motivations may overrule redistributional motivations. This is most likely to be the case when ability differences are not large.[19]

19 This result is in line with Lau and Poutvaara (2001), who report that without ability differences, Bismarckian old-age benefit systems lead to highest welfare and Beveridgean retirement subsidies to lowest welfare, while the ordering between the two remaining systems may vary.

**Table 5**
*Welfare Ordering of Social Security Systems when $\mu = 0.3$ and $\alpha = 0.96$*

| | | $d = 0$ | $d = 0.1$ | $d = 0.5$ | $d = 1.0$ |
|---|---|---|---|---|---|
| Low ability | 1. | BI_OA | BI_OA | BE_OA | BE_OA |
| | 2. | BE_OA | BE_OA | BI_OA | BE_RE |
| | 3. | BI_RE | BI_RE | BE_RE | BI_OA |
| | 4. | BE_RE | BE_RE | BI_RE | BI_RE |
| High ability | 1. | BI_OA | BI_OA | BI_OA | BI_OA |
| | 2. | BE_OA | BE_OA | BE_OA | BE_OA |
| | 3. | BI_RE | BI_RE | BI_RE | BI_RE |
| | 4. | BE_RE | BE_RE | BE_RE | BE_RE |
| Utilitarian measure | 1. | BI_OA | BI_OA | BI_OA | BI_OA |
| | 2. | BE_OA | BE_OA | BE_OA | BE_OA |
| | 3. | BI_RE | BI_RE | BI_RE | BI_RE |
| | 4. | BE_RE | BE_RE | BE_RE | BE_RE |

Low-ability types may even in some cases prefer the Bismarckian retirement subsidy system to the Beveridgean old-age benefit system. This situation occurs when the marginal utility of leisure is close to constant, which implies that retirement from the labor market is more abrupt. These results depend crucially on the endogenous human-capital formation in the model. A model with exogenous productivity profiles would suggest that low-ability types always prefer Beveridgean systems to Bismarckian systems, even when ability differences are small. Those models could therefore give a misleading picture of what social security system it would be in the interest of low-ability workers to opt for.[20]

Tables 5–7 report the welfare ordering of the four different systems for both ability types with different parameter combinations. The ordering of the different social security systems according to the utilitarian criterion is also included. Arrows mark the regions in which the preference ordering between different systems changes.

The utilitarian measure coincides with the preference ordering of high-ability types when ability differences are small, and it coincides with the

20 Our analysis assumes that there are no credit constraints. Therefore, a natural extension would be the role of credit constraints in the preference ordering of the alternative systems across different ability types. The importance of these considerations is highlighted by Casamatta et al. (2000), who show that with borrowing constraints, the retirees are supported by the middle-income workers, instead of those with the lowest income, in the maintenance of the social security system.

**Table 6**
*Welfare Ordering of Social Security Systems when $\mu = 0.1$ and $\alpha = 0.74$*

| | | $d = 0$ | $d = 0.1$ | $d = 0.5$ | $d = 1.0$ |
|---|---|---|---|---|---|
| Low ability | 1. | BI_OA | BI_OA | BE_OA | BE_OA |
| | 2. | BE_OA | BE_OA | BE_RE | BE_RE |
| | 3. | BI_RE | BI_RE | BI_OA | BI_OA |
| | 4. | BE_RE | BE_RE | BI_RE | BI_RE |
| High ability | 1. | BI_OA | BI_OA | BI_OA | BI_OA |
| | 2. | BE_OA | BI_RE | BI_RE | BI_RE |
| | 3. | BI_RE | BE_OA | BE_OA | BE_OA |
| | 4. | BE_RE | BE_RE | BE_RE | BE_RE |
| Utilitarian measure | 1. | BI_OA | BI_OA | BI_OA | BI_OA |
| | 2. | BE_OA | BE_OA | BE_OA | BE_OA |
| | 3. | BI_RE | BI_RE | BI_RE | BI_RE |
| | 4. | BE_RE | BE_RE | BE_RE | BE_RE |

**Table 7**
*Welfare Ordering of Social Security Systems when $\mu = 0.05$ and $\alpha = 0.70$*

| | | $d = 0$ | $d = 0.1$ | $d = 0.5$ | $d = 1.0$ |
|---|---|---|---|---|---|
| Low ability | 1. | BI_OA | BE_OA | BE_OA | BE_OA |
| | 2. | BE_OA | BI_OA | BE_RE | BE_RE |
| | 3. | BI_RE | BE_RE | BI_OA | BI_OA |
| | 4. | BE_RE | BI_RE | BI_RE | BI_RE |
| High ability | 1. | BI_OA | BI_OA | BI_OA | BI_OA |
| | 2. | BE_OA | BI_RE | BI_RE | BI_RE |
| | 3. | BI_RE | BE_OA | BE_OA | BE_OA |
| | 4. | BE_RE | BE_RE | BE_RE | BE_RE |
| Utilitarian measure | 1. | BI_OA | BI_OA | BI_OA | BI_OA |
| | 2. | BE_OA | BE_OA | BE_OA | BE_OA |
| | 3. | BI_RE | BI_RE | BI_RE | BI_RE |
| | 4. | BE_RE | BE_RE | BE_RE | BE_RE |

preference ordering of low-ability types when ability differences are large. Hence, the utilitarian social planner prefers Bismarckian systems to Beveridgean systems when the labor force is homogeneous, and Beveridgean systems to Bismarckian systems when the labor force is more heterogeneous. Since the Rawlsian social planner chooses the system preferred by the least well-off citizen, our analysis shows that the Rawlsian and utilitarian criteria may disagree on the ordering of the different social security systems. These results suggest that changes in the wage distribution may result in changes in the political support for different social security systems.

### 4.2.2. Rates of Return

In order to better understand what drives our results, we have calculated the rates of return offered by the four different systems to the different ability types. The internal rate of return offered by the social security system is the discount rate at which the net present value of benefits is equal to the net present value of contributions. For ability type $i$, $i \in$ {high, low}, the internal rate of return $\lambda_i$ is given by

$$\sum_{t=45}^{59} \frac{1}{(1+\lambda_i)^t} \cdot b_{i,t} = \tau_s \sum_{t=0}^{59} \frac{W}{(1+\lambda_i)^t} \cdot l_{i,t}.$$

Since we analyze a steady-state economy without technological progress or population growth, the sums of lifetime contributions and benefits are equal for each cohort, as well as for the cross section of all cohorts in any period. Therefore, the internal rate of return offered by the social security systems is equal to zero for each cohort in each system, and it would be equal to zero for the representative individual in the absence of ability differences. This implies that if we find a positive rate of return offered to one ability type, then the rate of return has to be negative for the other ability type, and the ability type with a negative rate of return is subsidizing the type with a positive rate of return.

Our results show a consistent pattern: Beveridgean systems imply intragenerational redistribution from high-ability types to low-ability types, whereas Bismarckian systems imply intragenerational redistribution from low-ability types to high-ability types. While the results on the direction of intragenerational redistribution in Beveridgean systems are fully in line with the very nature of such systems, the result that Bismarckian systems redistribute income from low-ability types to high-ability types is surprising and against the very idea that Bismarckian systems are intragenerationally fair. It is worth highlighting that our results arise when benefits depend on past contributions in a linear manner and the market interest rate is used in the present-value calculations. These results would not arise in a model

**Table 8**

*Internal Rates of Return (percentage points)*

| | Beveridgean old-age benefits | | | Beveridgean retirement subsidies | | | Bismarckian old-age benefits | | | Bismarckian retirement subsidies | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 |
| $\mu = 0.3$; $\alpha = 0.96$ | | | | | | | | | | | | |
| Low ability | 0.12 | 0.57 | 1.06 | 0.15 | 0.66 | 1.21 | −0.04 | −0.20 | −0.40 | −0.01 | −0.04 | −0.07 |
| High ability | −0.12 | −0.47 | −0.74 | −0.14 | −0.55 | −0.88 | 0.04 | 0.13 | 0.19 | 0.01 | 0.03 | 0.04 |
| $\mu = 0.1$; $\alpha = 0.74$ | | | | | | | | | | | | |
| Low ability | 0.12 | 0.57 | 1.05 | 0.15 | 0.69 | 1.25 | −0.03 | −0.17 | −0.39 | 0.00 | −0.02 | −0.04 |
| High ability | −0.12 | −0.46 | −0.73 | −0.14 | −0.58 | −0.91 | 0.03 | 0.11 | 0.18 | 0.00 | 0.01 | 0.02 |
| $\mu = 0.05$; $\alpha = 0.70$ | | | | | | | | | | | | |
| Low ability | 0.12 | 0.57 | 1.05 | 0.15 | 0.70 | 1.25 | −0.02 | −0.16 | −0.39 | 0.00 | −0.01 | −0.04 |
| High ability | −0.12 | −0.46 | −0.73 | −0.15 | −0.58 | −0.91 | 0.02 | 0.11 | 0.18 | 0.00 | 0.01 | 0.02 |

with an exogenous specification of full retirement immediately after the entitlement age. In such a model, a Bismarckian system (in the way we have specified it) would result in zero internal rates of return for both ability types.

The explanation for the redistribution from low-income earners to high-income earners is evident from figures 3 and 4. High-income earners consume more leisure after the entitlement age than do low-income earners. Since benefits do not depend on contributions made after the entitlement age, high-income earners receive a higher rate of return on their contributions under both Bismarckian systems than low-income earners. Excluding the intragenerational redistribution in a Bismarckian old-age benefit system would thus require that social security taxes be levied on income earned before the entitlement age, but that would not be sufficient to eliminate the intragenerational redistribution in a Bismarckian retirement subsidy system. Furthermore, it may not be desirable for low-income earners to pay social security taxes only on earnings before the entitlement age, because the net present value of social security contributions needed to collect a given revenue would then increase.

Taken together, the results reveal that efficiency considerations may overrule equity considerations even for low-ability types. Although a Bismarckian system induces redistribution from the poor to the rich, the system is also associated with higher production and in some cases preferred by the low-ability types over Beveridgean systems, from which they would be net beneficiaries of intragenerational redistribution.

Independently of the direction of intragenerational redistribution, all social security systems impose a net burden on both ability types. This can be evaluated by calculating the income effects of the social security system from an accountant's perspective, that is, in the absence of behavioral responses. We calculate the rate of return of a social security system by the net present value of benefits divided by the net present value of contributions. This gives the gross rate of return, and subtracting 1 gives the net rate of return. Formally,

$$ror = \frac{NPV(\text{benefits})}{NPV(\text{contributions})} - 1 = \frac{\sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot b_{i,t}}{\tau_s \sum_{t=0}^{59} \frac{W}{(1+r)^t} \cdot l_{i,t}} - 1.$$

We report the net rates of return of the different social security systems in table 9. We find that even in the most favorable cases, low-ability types lose 47% of the net present value of their social security contributions, while high-ability types lose at least 64%. The maximum loss for high-ability types is 74% and for low-ability types 68%.[21]

21 In a growing economy, losses imposed by a social security system would be smaller.

**Table 9**
*Rates of Return*

| | Beveridgean old-age benefits | | | Beveridgean retirement subsidies | | | Bismarckian old-age benefits | | | Bismarckian retirement subsidies | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 | 0.1 | 0.5 | 1.0 |
| $\mu = 0.3$; $\alpha = 0.96$ | | | | | | | | | | | | |
| Low ability | −0.64 | −0.58 | −0.52 | −0.62 | −0.55 | −0.47 | −0.66 | −0.66 | −0.67 | −0.64 | −0.64 | −0.64 |
| High ability | −0.67 | −0.71 | −0.74 | −0.65 | −0.70 | −0.73 | −0.66 | −0.66 | −0.66 | −0.64 | −0.64 | −0.64 |
| $\mu = 0.1$; $\alpha = 0.74$ | | | | | | | | | | | | |
| Low ability | −0.64 | −0.59 | −0.52 | −0.64 | −0.57 | −0.48 | −0.66 | −0.67 | −0.68 | −0.66 | −0.66 | −0.66 |
| High ability | −0.67 | −0.71 | −0.74 | −0.67 | −0.71 | −0.74 | −0.66 | −0.66 | −0.66 | −0.66 | −0.66 | −0.66 |
| $\mu = 0.05$; $\alpha = 0.70$ | | | | | | | | | | | | |
| Low ability | −0.64 | −0.59 | −0.52 | −0.64 | −0.57 | −0.48 | −0.66 | −0.67 | −0.68 | −0.66 | −0.66 | −0.66 |
| High ability | −0.67 | −0.71 | −0.74 | −0.67 | −0.71 | −0.74 | −0.66 | −0.66 | −0.66 | −0.66 | −0.66 | −0.66 |

These losses reflect the implicit tax burden on young generations due to the pay-as-you-go nature of the social security system. Abolishing the pay-as-you-go system would require defaulting on promised contributions to older workers during the transition period, or converting the implicit debt of the pay-as-you-go system into explicit debt. The theoretical literature suggests that converting implicit debt into an explicit debt cannot generate a Pareto improvement if the pay-as-you-go system is intragenerationally efficient and if there are no market failures. The argument against the possibility of a Pareto-improving elimination of an intragenerationally fair pay-as-you-go system is presented succinctly by Fenge (1995). Brunner (1996) extends the efficiency result of Fenge to an economy with endogenous labor supply and heterogeneous households.

The importance of behavioral responses is revealed by comparing the ordering of systems in tables 5–7 with the magnitudes of the rates of return in table 9. For both ability types there are cases in which the preferred social security system is associated with a less favorable rate of return from an accountant's perspective.

Our result suggests that there might still be scope for Pareto improvement through better planning of the social security system. Implementing actuarial adjustment in the form of old-age benefits rather than retirement subsidies seems to be the most likely candidate for an acceptable policy change for all ability types. Furthermore, we find that a Bismarckian system may even be preferred by the low-ability types. However, investigating whether a Pareto improvement is indeed possible would require a full model of dynamic transition, an effort beyond the scope of this paper.

Though we focus on zero population growth, the insight of Aaron (1966) suggests how introducing population growth is to be expected to affect the results. Aaron (1966) shows that the rate of return on a given cohort's contributions under the pay-as-you-go system equals the economy's growth rate. If the population or productivity growth rate decreases, this can be expected to have qualitatively similar effects on the desirability of a pay-as-you-go system to those of an increase in the interest rate. In an earlier version of this paper, we assumed a 5% interest rate, and in this case the loss that a pay-as-you-go system implied for steady-state cohorts was even higher. The qualitative results concerning time allocation over the life cycle were similar. Also including mortality should leave the qualitative results unchanged, provided that there are actuarially fair annuity markets. Formally, mortality would enter as higher discounting of future utility.

## 5. Conclusion

We find that the macroeconomic response to changes in social security provisions may be quite substantial. In particular, retirement decisions are found to depend crucially on whether there is an implicit tax on working after the entitlement age in the form of lost benefits. While benefits conditioned on not working tend to cause very abrupt retirement around the entitlement age, old-age benefits paid to everyone above the entitlement age independently of labor-market status lead to a more gradual and drawn-out retirement pattern. It also matters whether the benefits maintain a constant replacement ratio or not. In fact, within the range of parameter values considered, we find that the highest production level is obtained by the Bismarckian old-age benefit system, whereas the lowest production level is obtained by the Beveridgean retirement subsidy system. Furthermore, we find that the losses caused by retirement subsidy rather than old-age benefit systems tend to be monotonically increasing in the difference between ability types. This result is relevant in view of the changes in the structure of wages and employment that have been observed in most industrialized countries in recent years.

Finally, we find a surprising preference ordering of alternative social security systems by low-ability types. Most notably, if the ability differences are small, low-ability workers may actually prefer the Bismarckian old-age benefit system to the Beveridgean old-age benefit system. In other words, the positive efficiency effects dominate the adverse redistributive effects that would follow a transition from a Beveridgean to a Bismarckian pension system.

From a political-economy point of view, our findings are important. They suggest that European social security systems should be reformed by replacing current retirement subsidies with old-age benefits paid independently of labor-market status, and that such a reform should be widely supported. At the same time, a certain convergence towards a Bismarckian model of social security might be acceptable also to those with lower than average incomes.

## 6. Appendix

It is useful to derive the first-order conditions for the intertemporal household maximization problem for each of the four social security systems that we consider in the analysis. We focus below on economic incentive effects implied by the four alternative social security systems. The ability index is suppressed below, since the maximization problem is identical for the two types of households.

### 6.1. Beveridgean Old-age Benefits

The representative household maximizes the present value of lifetime utility subject to an intertemporal budget constraint, the law of motion with respect to human capital, and a time endowment constraint. Using the same notation as before, each household maximizes overall utility, which is given by

$$U = \sum_{t=0}^{59} \frac{1}{(1+\rho)^t} \left( \frac{c_t^{1-\gamma}}{1-\gamma} + \alpha\left(v_t - \mu v_t^2\right) \right). \tag{21}$$

This formulation of the utility function assumes that the representative household's utility at time 0 is a weighted sum of flows of utility over the life cycle.

The intertemporal maximization problem is derived by forming the Lagrangian and differentiating with respect to the instrumental variables and the state variable. The intertemporal budget constraint is given by

$$\sum_{t=0}^{59} \frac{W}{(1+r)^t} \cdot h_t^\beta q_t + \sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \kappa \geq \sum_{t=0}^{59} \frac{1}{(1+r)^t} \cdot c_t, \tag{22}$$

where $\kappa$ is the constant social security payment per period to individuals above the entitlement age.[22]

The law of motion with respect to human capital is given by

$$(1-\delta^H)h_t + s_t^\eta \geq h_{t+1}. \tag{23}$$

The time endowment constraint is given by

$$e \geq v_t + q_t + s_t. \tag{24}$$

The first-order necessary conditions for the primal maximization problem can be interpreted as zero-profit conditions for corresponding production activities. Differentiating the Lagrangian with respect to consumption yields

$$\lambda \cdot \frac{1}{(1+r)^t} = \frac{1}{(1+\rho)^t} \cdot c_t^{-\gamma}, \tag{25}$$

where $\lambda$ is the shadow price of one unit of intertemporal consumption. The price of one unit of consumption at time $t$ (LHS) is equal to the marginal benefit of one unit of consumption at time $t$.

The demand for leisure over the life cycle is determined by

$$\varpi_t \geq \frac{1}{(1+\rho)^t} \cdot \alpha \cdot (1 - 2\mu v_t), \tag{26}$$

where $\varpi_t$ is the shadow price of one unit of time at time $t$. The demand for leisure is equal to 0 if the price of one unit of time is larger than the marginal

22 Note that $W$ denotes the net return to labor services. This, rather than the gross return to labor services, is the variable relevant to individual optimization.

benefit of one unit of leisure, and the demand for leisure is positive if this constraint is binding.

Differentiating the Lagrangian with respect to work effort gives the following first-order condition:

$$\varpi_t \geq \lambda \cdot \frac{W}{(1+r)^t} \cdot h_t^{\beta}. \tag{27}$$

This equation determines the work effort over the life cycle. Hence, the work effort is positive if the price of one unit of time is equal to the marginal product of one unit of labor. Note that the supply of labor services is a function of the work effort and the accumulated stock of human capital.

The first-order condition with respect to investment in human capital is given by

$$\varpi_t \geq \theta_{t+1}\eta s_t^{\eta-1}, \tag{28}$$

where $\theta_t$ is the shadow price of one unit of human capital at time $t$. The cost of spending one unit of time on learning in period $t$ is greater than or equal to the marginal product of learning times the price of one unit of human capital in period $t+1$.

Finally, the first-order condition with respect to human capital is given by

$$\theta_t \geq \lambda \cdot \frac{W}{(1+r)^t} \cdot \beta h_t^{\beta-1} q_t + \theta_{t+1}(1-\delta^H). \tag{29}$$

The cost of one unit of human capital in period $t$ on the LHS is greater than or equal to the benefit on the RHS. The first term on the RHS is the return to human capital in period $t$, and the second is the net present value of the remaining stock of human capital in period $t+1$.

## 6.2. Bismarckian Old-age Benefits

Under the Bismarckian old-age benefit system, the individual intertemporal budget constraint changes to

$$\sum_{t=0}^{59} \frac{W}{(1+r)^t} \cdot h_t^{\beta} q_t + \sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \pi \geq \sum_{t=0}^{59} \frac{1}{(1+r)^t} \cdot c_t, \tag{30}$$

where $\pi$ is the earnings-related social security payment per period to individuals after the entitlement age. Old-age benefits depend on the wage-income history during the period before the entitlement age and are determined by

$$\sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \pi = x \cdot \sum_{t=0}^{44} \frac{W}{(1+r)^t} \cdot h_t^{\beta} q_t, \tag{31}$$

where $x$ is a positive number, which is determined in accordance with the public budget constraint. This specification implies that the net present value of old-age benefits during the official retirement period is a constant fraction of the net present value of after-tax wage income before the entitlement age.

The first-order condition with respect to work effort changes to

$$\varpi_t \geq (\lambda + \varsigma x)\frac{W}{(1+r)^t} \cdot h_t^{\beta},$$

where $\varsigma$ is the Lagrange multiplier with respect to earnings-related benefits, which is positive when $t \leq 44$ and 0 otherwise. Hence, the return to work effort is greater during the first 44 years of the life cycle than under the Beveridgean old-age benefit system.

The first-order condition with respect to human capital changes to

$$\theta_t \geq (\lambda + \varsigma x)\frac{W}{(1+r)^t}\beta h_t^{\beta-1} q_t + \theta_{t+1}(1-\delta^H). \tag{32}$$

The positive relation between old-age benefits and wage income yields a higher return to human capital before the official retirement age.

### 6.3. Beveridgean Retirement Subsidies

Under the Beveridgean retirement subsidy system, the individual intertemporal budget constraint changes to

$$\sum_{t=0}^{59} \frac{W}{(1+r)^t} \cdot h_t^{\beta} q_t + \sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \omega v_t \geq \sum_{t=0}^{59} \frac{1}{(1+r)^t} \cdot c_t, \tag{33}$$

where $\omega$ is the retirement subsidy.

The first-order condition with respect to leisure now reads

$$\varpi_t \geq \frac{1}{(1+\rho)^t} \cdot \alpha(1-2\mu v_t) + \frac{1}{(1+r)^t} \cdot \omega. \tag{34}$$

Compared to the two old-age benefit systems, this specification yields a higher return to leisure after the official retirement age.

### 6.4. Bismarckian Retirement Subsidies

The Bismarckian retirement subsidy system includes a link between social security contributions and benefits, where benefits after the entitlement age depend linearly on the wage income before that age. The individual budget constraint is determined by

$$\sum_{t=0}^{59} \frac{W}{(1+r)^t} \cdot h_t^{\beta} q_t + \sum_{t=45}^{59} \frac{1}{(1+r)^t} \cdot \psi v_t \geq \sum_{t=0}^{59} \frac{1}{(1+r)^t} \cdot c_t, \tag{35}$$

where $\psi$ is the income-dependent subsidy to retirement after the entitlement age. The social security benefit stream is equal to a fraction of the net present value of net wage income before the entitlement age:

$$\psi = z \cdot \sum_{t=0}^{44} \frac{W}{(1+r)^t} \cdot h_t^{\beta} q_t. \tag{36}$$

The first-order condition with respect to work effort is

$$\varpi_t \geq (\lambda + \xi z) \frac{W}{(1+r)^t} \cdot h_t^{\beta},$$

where $\xi$ is the Lagrange multiplier with respect to earnings-related benefits, which is positive when $t \leq 44$ and 0 otherwise. The first-order condition with respect to human capital changes to

$$\theta_t \geq (\lambda + \xi z) \frac{W}{(1+r)^t} \cdot \beta h_t^{\beta-1} q_t + \theta_{t+1}(1 - \delta^H). \tag{37}$$

The two first-order conditions with respect to work effort and human capital are similar to the corresponding first-order conditions under the Bismarckian old-age benefit system. Hence, the positive relation between old-age benefits and wage income yields a higher return to work effort and human capital before the official retirement age than do the two Beveridgean systems.

The first-order condition with respect to leisure is

$$\varpi_t \geq \frac{1}{(1+\rho)^t} \cdot \alpha(1 - 2\mu v_t) + \frac{1}{(1+r)^t} \cdot \psi. \tag{38}$$

This equation is similar to the corresponding first-order condition under the Beveridgean retirement subsidy system, and it yields a higher return to leisure after the official retirement age than do the two old-age benefit systems.

## References


Aaron, H. (1966), The Social Insurance Paradox, Canadian Journal of Economics and Political Science 33, 371–374.

Börsch-Supan, A. (2000), Incentive Effects of Social Security on Labor Force Participation, Journal of Public Economics 78(2), 25–49.

Brunner, J. (1996), Transition from a Pay-As-You-Go to a Fully Funded System: The Case of Differing Individuals and Intragenerational Fairness, Journal of Public Economics 60, 131–146.

Casamatta, G., Cremer, H., and Pestieau, P. (2000), The Political Economy of Social Security, Scandinavian Journal of Economics 102(3), 503–522.

Cremer, H., and Pestieau, P. (2003), The Double Dividend of Postponing Retirement, International Tax and Public Finance 10, 419–434.

Fenge, R. (1995), Pareto-Efficiency of the Pay-As-You-Go Pension System with Intragenerational Fairness, FinanzArchiv 52, 357–364.

Friedberg, L. (2003), The Impact of Technological Change on Older Workers: Evidence from Data on Computer Use, Industrial and Labor Relations Review 56(3), 511–529.

Gruber, J., and Wise, D.A. (1999), Introduction and Summary, in Social Security and Retirement Around the World. University of Chicago Press, Chicago, IL.

Harrison, G.W., Lau, M.I., and Williams, M.B. (2002), Estimating Individual Discount Rates in Denmark: A Field Experiment, American Economic Review 92, 1606–1617.
Heckman, J.J., Lochner, L., and Taber, C. (1998), Tax Policy and Human Capital Formation, American Economic Review, 88(2), 293–298.
Lau, M.I., Poutvaara, P., and Wagener, A. (2004), Dynamic Costs of the Draft, forthcoming in: German Economic Review.
Lau, M.I., and Poutvaara, P. (2001), Social Security Rules, Labor Supply and Human Capital Formation, CEBR Discussion Paper 2001-10, Centre for Economic and Business Research, Copenhagen.
McGratten, E.R., and Rogerson, R. (1998), Changes in Hours Worked Since 1950, Federal Reserve Bank of Minneapolis Quarterly Review 22, 2–19.
Perroni, C. (1995), Assessing the Dynamic Efficiency Gains of Tax Reform When Human Capital Is Endogenous, International Economic Review 36(4), 907–925.
Rutherford, T.F. (1999), Applied General Equilibrium Modeling with MPSGE as a GAMS Subsystem: An Overview of the Modeling Framework and Syntax, Computational Economics 14, 1–46.
Schmähl, W. (1993), Harmonization of Pension Schemes in Europe? A Controversial Issue in the Light of Economics, in: Atkinson, A.B., and Rein, M. (Eds.), Age, Work and Social Security, Macmillan, Houndsmills, London, 308–340.

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